Diagonally cyclic Latin squares

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A Latin square is a square matrix in which each row and column is a permutation of the same set of symbols. Examples include Cayley tables of finite groups and completed sudoku puzzles. A Latin square is diagonally cyclic if the symbols occur in cyclic order along each broken diagonal parallel to the main diagonal. An example of order 7, with one of its cyclic diagonals highlighted, is

$$\begin{bmatrix} 0 & 2 & 5 & 1 & 6 & 4 & 3 \\ 4 & 1 & 3 & 6 & 2 & 0 & 5 \\ 6 & 5 & 2 & 4 & 0 & 3 & 1 \\ 2 & 0 & 6 & 3 & 5 & 1 & 4 \\ 5 & 3 & 1 & 0 & 4 & 6 & 2 \\ 3 & 6 & 4 & 2 & 1 & 5 & 0 \\ 1 & 4 & 0 & 5 & 3 & 2 & 6 \end{bmatrix}$$

An orthomorphism of an abelian group G is a permutation $\theta : G \mapsto G$ such that the map $x \mapsto \theta(x) - x$ is also a permutation of G. It is not hard to find a bijection between diagonally cyclic Latin squares and orthomorphisms of cyclic groups. I will review the history and applications of diagonally cyclic Latin squares and orthomorphisms, including reporting new results of two current projects of mine, one of which is joint with Aleš Drápal (Charles University, Prague) and the other is joint work with my student Jack Allsop.