On Laplacian eigenvalues of graphs

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ABSTRACT

Let $G(V,E)$ be a simple graph of order n, size m and having the vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$ and edge set $E(G) = \{e_1, e_2, \dots, e_m\}$. The adjacency matrix $A = (a_{ij})$ of G is a $(0,1)$-square matrix of order n whose (i,j)-entry is equal to 1 if v_i is adjacent to v_j and equal to 0, otherwise. Let $D(G) = diag(d_1, d_2, \dots, d_n)$ be the diagonal matrix associated to G, where $d_i = \deg(v_i)$, for all $i = 1, 2, \dots, n$. The matrix $L(G) = D(G) - A(G)$ is called the Laplacian matrix and its eigenvalues are called the Laplacian eigenvalues of the graph G. We discuss about the Laplacian eigenvalues of certain family of graphs. Let $0 = \mu_n \leq \mu_{n-1} \leq \cdots \leq \mu_1$ be the Laplacian eigenvalues of G and $S_k(G) = \sum\limits_{i=1}^{k} \mu_i$, $k = 1, 2, \dots, n$ be the sum of k largest Laplacian eigenvalues of G. For any k, $k = 1, 2, \dots, n$, A. Brouwer conjectured that $S_k(G) = \sum\limits_{i=1}^{k} \mu_i \leq m + \binom{k+1}{2}$. We discuss the bounds for $S_k(G)$ and the recent developments of the Brouwer's conjecture. Further, we investigate analogous conjectures (of the Brouwer's type) in other types of graphs.